A NOTE ON THE THEORY OF THE COMBUSTION STABILITY OF POWDERS

S. S. Novikov and Yu. S. Ryazantsev

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 3, pp. 137-139, 1966

The combustion stability of powders was investigated in [1-5]. These authors [3-5] made use of the method of elementary wave solutions, which is standard in the theory of hydrodynamic stability.

It is known that this method does not always give an exhaustive investigation of stability [6]. Articles [3-5] also left the question of the completeness of the results and the physical sense of the obtained solutions partially unclarified.

In this note, the results of [3-5] are substantiated by considering the problem of changes in the steady-state mode of combustion of powders under the influence of a given disturbance. The problem is solved by the use of Laplace transforms.

It is assumed that disruption of the steady-state mode of combustion is caused by a slight change in the pressure $\delta p(t)$:

$$\delta p = \begin{cases} 0 & (t < 0), \\ -p_0 e^{-\sigma_0 t} & (t > 0) \quad \operatorname{Re} \sigma_0 > 0. \end{cases} \tag{0.1}$$

It can be shown that the results obtained below are not connected with the type of pressure disturbance (0.1). These results are also not changed if we assume that the disturbance in the steady-state mode is not caused by a change in pressure, but by a slight initial deviation in the temperature from the steady-state profile.

1. Let us consider a case in which the burning of a powder is described by the theory of Ya. B. Zel'dovich.

According to [2], a disturbance in the burning rate δU with small deviations from the steady-state mode depends only on disturbances in the pressure δp and the temperature gradient on the surface of the K-phase (condensed phase) $\delta\varphi$,

$$\frac{\delta U}{U} = \frac{\varepsilon}{\varepsilon - 1}\frac{\delta\varphi}{\varphi} + \frac{\nu}{1-\varepsilon}\frac{\delta p}{p},$$

$$\varphi = \frac{U}{\varkappa}(T_s - T_0), \quad \varepsilon = \left(\frac{\partial \ln U}{\partial T_0}\right)_p (T_s - T_0), \quad \nu = \left(\frac{\partial \ln U}{\partial \ln p}\right)_{T_0}. \tag{1.1}$$

Here U is the burning rate, $\varkappa$ is the coefficient of thermal conductivity, T_s is the temperature of the surface of the powder, p is the pressure, φ is the temperature gradient on the surface of the powder, and T_0 is the initial temperature.

Small deviations in the temperature $\tau(x, t)$ of the powder from the steady-state Michelson profile satisfy the linearlized heat conduction equation, which is of the following form in a coordinate system tied to the surface of the powder (x = 0):

$$\varkappa\frac{\partial^2\tau}{\partial x^2} - U\frac{\partial\tau}{\partial x} - \frac{\partial\tau}{\partial t} = \delta U\,\frac{U}{\varkappa}(T_s - T_0)\exp\frac{Ux}{\varkappa}. \tag{1.2}$$

The following boundary conditions follow from the conditions of constancy of the temperature on the burning surface and at a great distance from it:

$$\tau(0, t) = \tau_s = 0, \qquad \tau(-\infty, t) = 0. \tag{1.3}$$

Stationarity of the undisturbed mode of combustion implies the initial condition

$$\tau(x, 0) = 0 \qquad -\infty < x < 0. \tag{1.4}$$

We shall find the solution of the problem (1.1)-(1.4) by using Laplace transforms, introducing the transformed temperature by means of the formula

$$\tau^*(x, \sigma) = \int\limits_0^\infty e^{-\sigma t}\tau(x, t)\,dt. \tag{1.5}$$

From Eqs. (1.2) and conditions (0.1), (1.1), and (1.3), following use of Laplace transforms, we obtain

$$\tau^*(x, \sigma) = \frac{T_s - T_0}{\varphi s}\left(\exp\alpha_1 x - \exp\frac{Ux}{\varkappa}\right)\left[\frac{1}{\varepsilon - 1}\left(\frac{d\tau^*}{dx}\right)_0 + \right.$$
$$\left. + \frac{\varphi}{p}\frac{\nu}{\varepsilon - 1}\frac{p_0}{\sigma + \sigma_0}\right] \tag{1.6}$$
$$\left(\alpha_1 = \frac{U}{2\varkappa}(1 + \sqrt{1 + 4s}), \quad s = \frac{\sigma\varkappa}{U^2}\right).$$

Differentiating (1.6) and setting x = 0, we find that

$$\left(\frac{d\tau^*}{dx}\right)_0 =$$
$$= \frac{\nu p_0 (T_s - T_0)(1 - \sqrt{1 + 4s})\,[2s(1-\varepsilon) - \varepsilon - \varepsilon\sqrt{1 + 4s}]}{4pUs(s + s_0)\,[(1-\varepsilon)^2 s - \varepsilon]}. \tag{1.7}$$

Taking (1.7) into consideration, formula (1.6) yields an explicit expression for a disturbance in the temperature of the powder transformed by Laplace transforms.

By applying the inversion formula to the function $\tau^*(x, \delta)$, we can derive an expression for the function $\tau(x, t)$ describing the change in the distribution of temperatures in the K-phase under the action of pressure disturbances $\delta p(t)$.

However, in order to investigate the stability of the steady-state mode of combustion, it is sufficient to study the expression for the temperature gradient on the burning surface, which is of the form

$$\frac{\partial\tau(0, t)}{\partial x} = \int\limits_{\gamma - i\infty}^{\gamma + i\infty} e^{\sigma t}\left(\frac{d\tau^*}{dx}\right)_0 d\sigma, \qquad \operatorname{Re}\gamma > 0. \tag{1.8}$$

The line of integration in (1.8) passes through the complex plane σ to the right of the singularities of the integrand, which are located at points $\sigma_1 = 0$ (poles), $\sigma_2 = \sigma_0$ (pole), $\sigma_3 = U^2\varepsilon/(1-\varepsilon)^2$ (pole), and $\sigma_4 = -U^2/4\varkappa$ (branch point).

Making use of the standard method of calculating the original from transforms [7], we conclude that the integral (1.8) is equal to the sum of the residues of the integrand at the poles $\sigma_{1,2,3}$ and the integral along the contours of the section on the complex plane σ, which is conveniently drawn from the branch point σ_4 along the real axis to $\sigma \to -\infty$. The residue of the integrand is zero at point σ_1. The residue at point σ_2 contributes to the form of the time-dependent term as $\exp(-\sigma_0 t)$.

The residue of the integrand at point σ_2 and the integral along the contours of the section decrease exponentially in time, while the residue at point σ_3 is equal to zero when $\varepsilon < 1$, but is nonzero and depends on time as $\exp[U^2\varepsilon t/\varkappa(1-\varepsilon)^2]$ when $\varepsilon > 1$.

It follows from this that when $\varepsilon < 1$, disturbances in the mode of combustion caused by changes in the pressure δp are attenuated with time. When $t \to \infty$, the temperature distribution in the powder and the burning rate become stationary values and combustion is stable.

When $\varepsilon > 1$, disturbances in the burning rate and the temperature are not attenuated with increasing t. The dependence of disturbances on time asymptotically approach an exponential relationship of the form $\exp[U^2\varepsilon t/\varkappa(1-\varepsilon)^2]$, and combustion is unstable.

2. Let us examine the case in which small deviations from the steady-state mode of combustion caused by disturbances pressure (0.1) change not only the burning rate and the temperature gradient on the surface of the powder, but also the temperature at the surface [3-5].

The relationships between pressure disturbances, burning rate, temperature gradient, and the temperature on the surface of the powder can be written in the form

$$\frac{\delta U}{U} = a_1 \frac{\delta\varphi}{\varphi} + b_1 \frac{\delta p}{p}, \qquad \frac{\tau_s}{T_s - T_0} = a_2 \frac{\delta\varphi}{\varphi} + b_2 \frac{\delta p}{p}. \quad (2.1)$$

In [8], formulas (2.1) were obtained on the assumption of the validity of the steady-state relationships $U(p, \varphi)$ and $T_s(p, \varphi)$ under unsteady conditions.

The following notation is introduced in the relations of (2.1):

$$a_1 = \frac{\varepsilon}{\varepsilon + r - 1}, \qquad b_1 = \frac{\nu(r-1) - \mu\varepsilon}{\varepsilon + r - 1}, \qquad a_2 = \frac{r}{\varepsilon + r - 1},$$

$$b_2 = \frac{\mu(\varepsilon - 1) - \nu r}{\varepsilon + r - 1}, \quad r = \left(\frac{\partial T_s}{\partial T_0}\right)_p, \quad \mu = \frac{1}{T_s - T_0}\left(\frac{\partial T_s}{\partial \ln p}\right)_{T_0}. \quad (2.2)$$

The relations of (2.1) also follow from taking approximate account of the chemical reaction of decomposition in the powder [4].

In the case under consideration, the behavior of small disturbances in the burning rate δU and the temperature of the powder $\tau(x, t)$ are determined from (1.2) with conditions (2.1) and (1.4).

As before, the solution of the problem can be obtained by using Laplace transforms, and the stability investigation is reduced to studying the expression for the disturbance of the temperature gradient on the surface of the powder, after conversion to Laplace transforms, which is of the form

$$\left(\frac{d\tau^*}{dx}\right)_0 = \frac{\nu p_0 (T_s - T_0)}{4pUs(s + s_0)(s - s_1)(s - s_2)} [(\mu\varepsilon - \nu r + \nu)(1 -$$

$$- \sqrt{1 + 4s}) + s(\mu\varepsilon - \mu - \nu r) \times$$

$$\times (1 + \sqrt{1 + 4s})] [(rs + \varepsilon)\sqrt{1 + 4s} + \varepsilon - s(2 - 2\varepsilon - r)],$$

$$s_{1,2} = \frac{(\varepsilon - 1)^2 - r(\varepsilon + 1) \pm [((\varepsilon - 1)^2 - r(\varepsilon + 1))^2 - 4r^2\varepsilon]^{1/2}}{2r^2}. \quad (2.3)$$

The quantity $(\partial\tau/\partial x)_0$ determined from formulas (1.8) and (2.3) is equal to the sum of the residues of the integrand at the poles $\sigma_1 = 0$, $\sigma_2 = \sigma_0$, $\sigma_3 = U^2 s_1/\varkappa$, $\sigma_4 = U^2 s_2/\varkappa$ and the integral along the contours of the section along the real axis from the branch point $\sigma_5 = -U^2/4\varkappa$ to $\sigma = -\infty$.

The residue at pole σ_1 is equal to zero. The residue at pole σ_2 and the integral along the section tend exponentially toward zero with increasing t. The values of the residues of the integrand at poles σ_4 and σ_5 depend materially on the values of ε and r. It is known from experiments that the real values of ε and r lie in the interval $\varepsilon > 0$ $(0 < r < 1)$.

When $\varepsilon < 1$, $r < (\varepsilon - 1)^2/(\varepsilon + 1)$, we can verify through direct computations that the residues at poles σ_3, σ_4 are equal to zero. The disturbances in the steady-state mode damp out with time. The combustion is stable.

If $\varepsilon < 1$, the residues at poles σ_3 and σ_4 are nonzero and depend on time as $\exp(s_1 U^2 t/\varkappa)$ and $\exp(s_2 U^2 t/\varkappa)$, respectively.

When $\varepsilon < 1$ and $r < (\varepsilon - 1)^2/(\varepsilon + 1)$, the values of the residues at poles σ_3 and σ_4 increase exponentially with increases in time, and the mode of combustion is unstable.

When $\varepsilon > 1$ and $r > (\varepsilon - 1)^2/(\varepsilon + 1)$, the values of the residues at σ_3 and σ_4 decrease exponentially with increasing t. The mode of combustion is stable. When $\varepsilon > 1$ and $r = (\varepsilon - 1)^2/(\varepsilon + 1)$, the residues at points σ_3 and σ_4 include a time factor of the form $\exp[it(\varepsilon)^{1/2}/r]$ (the boundary of instability).

It follows that the analysis presented in [3–5] of the combustion stability of powder is complete within the framework of the quasi-stationary description of the combustion zone adopted in these references. The sustained exponential solutions found in [3–5] describe the asymptotic behavior of small disturbances of a steady-state burning rate and steady-state temperature distribution.

The authors wish to thank A. G. Istratov, V. B. Librovich, and A. I. Leonov for their comments.

REFERENCES

1. Ya. B. Zel'dovich, "An approach to the theory of combustion of powders and explosives," ZhETF, vol. 12, 498, 1942.

2. Ya. B. Zel'dovich, "The burning rates of powders at changing pressures," PMTF, no. 3, 1964.

3. A. G. Istratov and V. B. Librovich, "The combustion stability of powders," PMTF, no. 5, 1964.

4. S. S. Novikov and Yu. S. Ryazantsev, "On the theory of combustion stability of powders," PMTF [Journal of Applied Mechanics and Technical Physics], no. 1, 1965.

5. B. V. Novozhilov, "Stability criterion for steady-state burning of powders," PMTF [Journal of Applied Mechanics and Technical Physics], no. 4, 1965.

6. K. M. Keiz, "Hydrodynamic stability as a problem with initial data," in: Hydrodynamic Instability [in Russian], Moscow, Izd-vo "Mir," 1964.

7. G. Doetsch, A Guide to Practical Applications of Laplace Transforms [Russian translation], Fizmatgiz, 1960.

8. B. V. Novozhilov, "Burning of a powder under harmonically varying pressure," PMTF [Journal of Applied Mechanics and Technical Physics], no. 6, 1965.

10 November 1966 Moscow